Engineering Mechanics
STATICS

FOURTH EDITION

Andrew Pytel / Jaan Kiusalaas

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***Engineering Mechanics: Statics*, Fourth Edition**

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To Jean, Leslie, Lori, John, Nicholas

and

To Judy, Nicholas, Jennifer, Timothy

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* Indicates optional articles

PREFACE

Statics and dynamics courses form the foundation of engineering mechanics, a branch of engineering that is concerned with the behavior of bodies under the action of forces. Engineering mechanics plays a fundamental role in civil, mechanical, aerospace and architectural engineering. In addition, the principles of engineering mechanics are often applied in other engineering fields and also in areas outside engineering, such as chemistry, physics, medicine and biology.

The principles of statics and dynamics are relatively few in number. Application of these principles to real-world problems requires insight gained from experience, rather than memorization. Therefore, all engineering textbooks, including this one, contain a large number of problems that are to be solved by the student. Learning the engineering approach to problem solving is one of the more valuable lessons to be learned in an introductory statics course.

New to This Edition The following proven features have been retained from the previous edition with significant updates.

- Sample problems illustrate the concepts introduced in each section.
- The homework problems are balanced between "textbook" problems and problems related to practical applications.
- The number of problems using U.S. Customary units and SI units are approximately equal.
- The importance of free-body diagrams is emphasized throughout the text.
- Equilibrium analysis is introduced in three separate sections. The first section teaches the drawing of free-body diagrams. The second section shows how to derive the equilibrium equations from given free-body diagrams. The third section illustrates how the above skills can be used to develop a workable plan for complete analysis of an equilibrium problem.
- Whenever applicable, the number of independent equilibrium equations is compared to the number of unknowns before the equilibrium equations are written.
- Review problems at the end of each chapter are intended to give the student additional practice in identifying and solving the various types of problems covered in the chapter.

In addition, approximately 30% of the homework problems are either new or have been modified from the previous edition.

As is typical, our textbook contains more material than can be covered in a three credit course in statics. The topics that can be omitted without jeopardizing continuity are marked with an asterisk (*). The asterisk is also used to indicate problems that require advanced reasoning by the student. The icon representing a computer disk denotes problems that require the use of a computer for a complete solution.

Chapter 1 begins with a review of vectors and vector operations. Chapter 2, applies these vector operations to systems of forces, including moments of forces. Chapter 3 follows with a discussion of the resultants of both two-dimensional and three-dimensional force systems. Also included in this chapter is an introduction to distributed normal loading which includes the concepts of centroids of volumes and areas. Therefore, Chapters 1–3 provide the mathematical preliminaries necessary for the student to begin a study of equilibrium.

Chapter 4 considers the equilibrium analysis of coplanar force systems. The all-important concept of a free-body diagram is introduced. The construction of free-body diagrams is one of the more important skills to be mastered in statics because free-body diagrams play a role in virtually every engineering application that considers the effects of forces upon bodies. This chapter separates equilibrium analysis into three fundamental steps: construction of the free-body diagram, comparing the number of unknowns with the number of independent equilibrium equations, and then writing and solving these equations. The chapter concludes with the analysis of plane trusses.

Chapter 5 presents the analysis of three-dimensional equilibrium. Taken as a unit, Chapters 4 and 5 contain a unified and complete discussion of the fundamental principles of static equilibrium. Therefore, the student is now prepared to apply these principles to several special applications of interest to engineers.

Chapter 6 is an optional chapter that is devoted to analysis of beams and cables. These two topics are fundamental to design in a course in mechanics of materials. However, these topics are included in most statics texts because they represent interesting applications of equilibrium analysis to practical engineering problems.

Chapter 7 presents the fundamentals of dry friction Chapter 7. In addition to the theory, the methods for analyzing equilibrium problems involving dry friction are also completely outlined. The analysis of friction problems will provide the student with additional practice in drawing free-body diagrams and writing equilibrium equations. Also included in this chapter are optional discussions of several applications of engineering interest, specifically, ropes and flat belts, disk friction and rolling resistance.

Chapter 8 discusses centroids and their relationships to simple and complex distributed loads are Chapter 8. (Recall that the concept of a centroid was first introduced in Chapter 3 and then used throughout equilibrium analysis wherever appropriate.) In addition to discussing the centroids of distributed normal loads, Chapter 8 also considers the centroids of plane and curved surfaces, and the centers of mass and gravity.

Chapter 9 considers the moment of inertia of area, also known as the second moment of area, which plays an important role in the study of the mechanics of materials. This subject is traditionally included in a statics text because it belongs to the progression of topics: area, first moment of area and second moment of area.

Chapter 10 shows how to use work-energy principles instead of Newton's laws to solve equilibrium problems. This is considered an optional chapter because there is often not enough time for the subject to be covered in the first statics course. However, if time permits, studying Chapter 10 gives the student a useful introduction and insight into the work-energy method that is often used in dynamics.

The Sample problems that require numerical solutions have been solved using MATLAB®.

Student's Website *Study Guide for Pytel and Kiusalaas's Engineering Mechanic, Statics, Fourth Edition,* J. L. Pytel and A. Pytel. The study guide includes self-tests to help the students focus on the important features of each chapter. Guided problems give students an opportunity to work through representative problems before attempting to solve the problems in the text.

Instructor's Website A detailed *Instructor's Solutions Manual* and *Lecture Note PowerPoints* slides, are available for instructors through a password protected Web site at www.cengagebrain.com.

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They have skillfully guided every aspect of this text's development and production to successful completion.

ANDREW PYTEL
JAAN KIUSALAAS

Introduction to Statics 1

1.1 Introduction

1.1a What is engineering mechanics?

Mechanics is the branch of physics that considers the action of forces on bodies or fluids that are *at rest* or *in motion*. Correspondingly, the primary topics of mechanics are statics and dynamics. *Engineering mechanics* is the branch of engineering that applies the principles of mechanics to mechanical design (i.e., any design that must take into account the effect of forces).

Engineering mechanics is an integral component of the education of engineers whose disciplines are related to the mechanical sciences, such as aerospace engineering, architectural engineering, civil engineering, and mechanical engineering.

Science Source

The Flemish mathematician and engineer Simon Stevinus (1548–1620) was the first to demonstrate resolution of forces, thereby establishing the foundation of modern statics.

1.1b Problem formulation and the accuracy of solutions

Your mastery of the principles of engineering mechanics will be reflected in your ability to formulate and solve problems. Unfortunately, there is no simple method for teaching problem-solving skills. Nearly all individuals require a considerable amount of practice in solving problems before they begin to develop the analytical skills that are so necessary for success in engineering. For this reason, a relatively large number of sample problems and homework problems are placed at strategic points throughout this text.

To help you develop an "engineering approach" to problem analysis, you will find it instructive to divide your solution for each problem into the following parts:

1. GIVEN: After carefully reading the problem statement, list all the data provided. If a figure is required, sketch it neatly and approximately to scale.
2. FIND: State the information that is to be determined.
3. SOLUTION: Solve the problem, showing all the steps that you used in the analysis. Work neatly so that your work can be followed by others.
4. VALIDATE: Many times, an invalid solution can be uncovered by simply asking yourself, "Does the answer make sense?"

When reporting your answers, use only as many digits as in the given data. For example, suppose that you are required to convert 12 500 ft (assumed to be accurate to three significant digits) to miles. Using a calculator, you would divide 12 500 ft by 5280 ft/mi and report the answer as 2.37 mi (three significant digits), although the quotient displayed on the calculator would be 2.367 424 2. Reporting the answer as 2.367 424 2 implies that all eight digits are significant, which is, of course, untrue. It is your responsibility to round off the answer to the correct number of digits. *In this text*, you should assume that given data are accurate to three significant digits unless stated otherwise. For example, a length that is given as 3 ft should be interpreted as 3.00 ft.

When performing intermediate calculations, a good rule of thumb is to carry one more digit than will be reported in the final answer; for example, use four-digit intermediate values if the answer is to be significant to three digits.

Furthermore, it is common practice to report four digits if the first digit in an answer is 1; for example, use 1.392 rather than 1.39.

1.2 Newtonian Mechanics

1.2a Scope of Newtonian mechanics

In 1687 Sir Isaac Newton (1642–1727) published his celebrated laws of motion in *Principia (Mathematical Principles of Natural Philosophy)*. Without a doubt, this work ranks among the most influential scientific books ever published. We should not think, however, that its publication immediately established classical mechanics. Newton's work on mechanics dealt primarily with celestial mechanics and was thus limited to particle motion. Another two hundred or so years elapsed before rigid-body dynamics, fluid mechanics, and the mechanics of deformable bodies were developed. Each of these areas required new axioms before it could assume a usable form.

Nevertheless, Newton's work is the foundation of classical, or Newtonian, mechanics. His efforts have even influenced two other branches of mechanics, born at the beginning of the twentieth century: relativistic and quantum mechanics. *Relativistic mechanics* addresses phenomena that occur on a cosmic scale (velocities approaching the speed of light, strong gravitational fields, etc.). It removes two of the most objectionable postulates of Newtonian mechanics: the existence of a fixed or inertial reference frame and the assumption that time is an absolute variable, "running" at the same rate in all parts of the universe. (There is evidence that Newton himself was bothered by these two postulates.) *Quantum mechanics* is concerned with particles on the atomic or subatomic scale. It also removes two cherished concepts of classical mechanics: determinism and continuity. Quantum mechanics is essentially a probabilistic theory; instead of predicting an event, it determines the likelihood that an event will occur. Moreover, according to this theory, the events occur in discrete steps (called *quanta*) rather than in a continuous manner.

Relativistic and quantum mechanics, however, have by no means invalidated the principles of Newtonian mechanics. In the analysis of the motion of bodies encountered in our everyday experience, both theories converge on the equations of Newtonian mechanics. Thus the more esoteric theories actually reinforce the validity of Newton's laws of motion.

1.2b Newton's laws for particle motion

Using modern terminology, Newton's laws of particle motion may be stated as follows:

1. If a particle is at rest (or moving with constant velocity in a straight line), it will remain at rest (or continue to move with constant velocity in a straight line) unless acted upon by a force.
2. A particle acted upon by a force will accelerate in the direction of the force. The magnitude of the acceleration is proportional to the magnitude of the force and inversely proportional to the mass of the particle.
3. For every action, there is an equal and opposite reaction; that is, the forces of interaction between two particles are equal in magnitude and oppositely directed along the same line of action.

Although the first law is simply a special case of the second law, it is customary to state the first law separately because of its importance to the subject of statics.

1.2c Inertial reference frames

When applying Newton's second law, attention must be paid to the coordinate system in which the accelerations are measured. An *inertial reference frame* (also known as a Newtonian or Galilean reference frame) is defined to be any rigid coordinate system in which Newton's laws of particle motion relative to that frame are valid with an acceptable degree of accuracy. In most design applications used on the surface of the earth, an inertial frame can be approximated with sufficient accuracy by attaching the coordinate system to the earth. In the study of earth satellites, a coordinate system attached to the sun usually suffices. For interplanetary travel, it is necessary to use coordinate systems attached to the so-called fixed stars.

It can be shown that any frame that is translating with constant velocity relative to an inertial frame is itself an inertial frame. It is a common practice to omit the word *inertial* when referring to frames for which Newton's laws obviously apply.

1.2d Units and dimensions

The standards of measurement are called *units*. The term *dimension* refers to the type of measurement, regardless of the units used. For example, kilogram and feet/second are units, whereas mass and length/time are dimensions. Throughout this text we use two standards of measurement: U.S. Customary system and SI system (from *Système internationale d'unités*). In the *U.S. Customary system* the base (fundamental) dimensions* are force $[F]$, length $[L]$, and time $[T]$. The corresponding base units are pound (lb), foot (ft), and second (s). The base dimensions in the *SI system* are mass $[M]$, length $[L]$, and time $[T]$, and the base units are kilogram (kg), meter (m), and second (s). All other dimensions or units are combinations of the base quantities. For example, the dimension of velocity is $[L/T]$, the units being ft/s, m/s, and so on.

A system with the base dimensions $[FLT]$ (such as the U.S. Customary system) is called a *gravitational system*. If the base dimensions are $[MLT]$ (as in the SI system), the system is known as an *absolute system*. In each system of measurement, the base units are defined by physically reproducible phenomena or physical objects. For example, the second is defined by the duration of a specified number of radiation cycles in a certain isotope, the kilogram is defined as the mass of a certain block of metal kept near Paris, France, and so on.

All equations representing physical phenomena must be *dimensionally homogeneous;* that is, each term of an equation must have the same dimension. Otherwise, the equation will not make physical sense (it would be meaningless, for example, to add a force to a length). Checking equations for dimensional homogeneity is a good habit to learn, as it can reveal mistakes made during algebraic manipulations.

1.2e Mass, force, and weight

If a force **F** acts on a particle of mass m, Newton's second law states that

$$\mathbf{F} = m\mathbf{a} \tag{1.1}$$

*We follow the established custom and enclose dimensions in brackets.

where **a** is the acceleration vector of the particle. For a gravitational $[FLT]$ system, dimensional homogeneity of Eq. (1.1) requires the dimension of mass to be

$$[M] = \left[\frac{FT^2}{L}\right] \tag{1.2a}$$

In the U.S. Customary system, the derived unit of mass is called a *slug*. A slug is defined as the mass that is accelerated at the rate of 1.0 ft/s^2 by a force of 1.0 lb. Substituting units for dimensions in Eq. (1.2a), we get for the unit of a slug

$$1.0 \text{ slug} = 1.0 \text{ lb} \cdot \text{s}^2/\text{ft}$$

For an absolute $[MLT]$ system of units, dimensional homogeneity of Eq. (1.1) yields for the dimension of force

$$[F] = \left[\frac{ML}{T^2}\right] \tag{1.2b}$$

The derived unit of force in the SI system is a *newton* (N), defined as the force that accelerates a 1.0-kg mass at the rate of 1.0 m/s^2. From Eq. (1.2b), we obtain

$$1.0 \text{ N} = 1.0 \text{ kg} \cdot \text{m/s}^2$$

Weight is the force of gravitation acting on a body. Denoting gravitational acceleration (free-fall acceleration of the body) by g, the weight W of a body of mass m is given by Newton's second law as

$$W = mg \tag{1.3}$$

Note that mass is a constant property of a body, whereas weight is a variable that depends on the local value of g. The gravitational acceleration on the surface of the earth is approximately 32.2 ft/s^2, or 9.81 m/s^2. Thus the mass of a body that weighs 1.0 lb on earth is (1.0 lb)/(32.2 ft/s^2) = 1/32.2 slug. Similarly, if the mass of a body is 1.0 kg, its weight on earth is (9.81 m/s^2)(1.0 kg) = 9.81 N.

At one time, the pound was also used as a unit of mass. The *pound mass* (lbm) was defined as the mass of a body that weighs 1.0 lb on the surface of the earth. Although pound mass is an obsolete unit, it is still used occasionally, giving rise to confusion between mass and weight. In this text, we use the pound exclusively as a unit of force.

1.2f Conversion of units

A convenient method for converting a measurement from one set of units to another is to multiply the measurement by appropriate conversion factors. For example, to convert 240 mi/h into ft/s, we proceed as follows:

$$240 \text{ mi/h} = 240\frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \times \frac{1.0\,\cancel{\text{h}}}{3600 \text{ s}} \times \frac{5280 \text{ ft}}{1.0\,\cancel{\text{mi}}} = 352 \text{ ft/s}$$

where the multipliers 1.0 h/3600 s and 5280 ft/1.0 mi are conversion factors. Because 1.0 h = 3600 s and 5280 ft = 1.0 mi, we see that each conversion factor is dimensionless and of magnitude 1. Therefore, a measurement is unchanged when it is multiplied by conversion factors—only its units are altered. Note that it is permissible to cancel units during the conversion as if they were algebraic quantities.

Conversion factors applicable to mechanics are listed inside the front cover of the book.

1.2g Law of gravitation

In addition to his many other accomplishments, Newton also proposed the law of universal gravitation. Consider two particles of mass m_A and m_B that are separated by a distance R, as shown in Fig. 1.1. The law of gravitation states that the two particles are attracted to each other by forces of magnitude F that act along the line connecting the particles, where

$$F = G\frac{m_A m_B}{R^2} \qquad \textbf{1.4}$$

The universal gravitational constant G is approximately equal to 3.44×10^{-8} ft^4/(lb · s^4), or 6.67×10^{-11} m^3/(kg · s^2). Although this law is valid for particles, Newton showed that it is also applicable to spherical bodies, provided their masses are distributed uniformly. (When attempting to derive this result, Newton was forced to develop calculus.)

If we let $m_A = M_e$ (the mass of the earth), $m_B = m$ (the mass of a body), and $R = R_e$ (the mean radius of the earth), then F in Eq. (1.4) will be the weight W of the body. Comparing $W = GM_e m/R_e^2$ with $W = mg$, we find that $g = GM_e/R_e^2$. Of course, adjustments may be necessary in the value of g for some applications in order to account for local variation of the gravitational attraction.

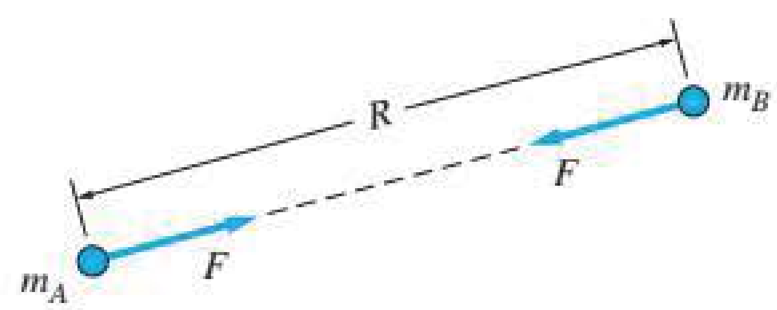


Figure 1.1

Sample Problem 1.1

Convert 5000 lb/in.^{2} to Pa (1 Pa = 1 N/m^2).

Solution

Using the conversion factors listed inside the front cover, we obtain

$$5000\ \text{lb/in.}^2 = 5000\frac{\cancel{\text{lb}}}{\cancel{\text{in.}}^2}\times\frac{4.448\ \text{N}}{1.0\ \cancel{\text{lb}}}\times\left(\frac{39.37\ \cancel{\text{in.}}}{1.0\ \text{m}}\right)^2$$

$$= 34.5\times10^6\ \text{N/m}^2 = 34.5\ \text{MPa} \qquad \textbf{Answer}$$

Sample Problem 1.2

The acceleration a of a particle is related to its velocity v, its position coordinate x, and time t by the equation

$$a = Ax^3t + Bvt^2 \qquad \text{(a)}$$

where A and B are constants. The dimension of the acceleration is length per unit time squared; that is, $[a] = [L/T^2]$. The dimensions of the other variables are $[v] = [L/T]$, $[x] = [L]$, and $[t] = [T]$. Derive the dimensions of A and B if Eq. (a) is to be dimensionally homogeneous.

Solution

For Eq. (a) to be dimensionally homogeneous, the dimension of each term on the right-hand side of the equation must be $[L/T^2]$, the same as the dimension for a. Therefore, the dimension of the first term on the right-hand side of Eq. (a) becomes

$$[Ax^3t] = [A][x^3][t] = [A][L^3][T] = \left[\frac{L}{T^2}\right] \qquad \text{(b)}$$

Solving Eq. (b) for the dimension of A, we find

$$[A] = \frac{1}{[L^3][T]}\left[\frac{L}{T^2}\right] = \frac{1}{[L^2T^3]} \qquad \textbf{Answer}$$

Performing a similar dimensional analysis on the second term on the right-hand side of Eq. (a) gives

$$[Bvt^2] = [B][v][t^2] = [B]\left[\frac{L}{T}\right][T^2] = \left[\frac{L}{T^2}\right] \qquad \text{(c)}$$

Solving Eq. (c) for the dimension of B, we find

$$[B] = \left[\frac{L}{T^2}\right]\left[\frac{T}{L}\right]\left[\frac{1}{T^2}\right] = \left[\frac{1}{T^3}\right] \qquad \textbf{Answer}$$

Sample Problem 1.3

Find the gravitational force exerted by the earth on a 70-kg man whose elevation above the surface of the earth equals the radius of the earth. The mass and radius of the earth are $M_e = 5.9742 \times 10^{24}$ kg and $R_e = 6378$ km, respectively.

Solution

Consider a body of mass m located at the distance $2R_e$ from the center of the earth (of mass M_e). The law of universal gravitation, from Eq. (1.4), states that the body is attracted to the earth by the force F given by

$$F = G\frac{mM_e}{(2R_e)^2}$$

where $G = 6.67 \times 10^{-11}\ \text{m}^3/(\text{kg} \cdot \text{s}^2)$ is the universal gravitational constant. Substituting the values for G and the given parameters, the earth's gravitational force acting on the 70-kg man is

$$F = (6.67 \times 10^{-11})\frac{(70)(5.9742 \times 10^{24})}{[2(6378 \times 10^3)]^2} = 171.4\ \text{N} \qquad \textbf{Answer}$$

PROBLEMS

1.1 A person weighs 30 lb on the moon, where $g = 5.32$ ft/s^2. Determine (a) the mass of the person; and (b) the weight of the person on earth.

1.2 The radius and length of a steel cylinder are 40 mm and 110 mm, respectively. If the mass density of steel is 7850 kg/m^3, determine the weight of the cylinder in pounds.

1.3 Convert the following: (a) 400 lb · ft to kN · m; (b) 6 m/s to mi/h; (c) 20 lb/in.^{2} to kPa; and (d) 500 slug/in. to kg/m.

1.4 A compact car travels 30 mi on one gallon of gas. Determine the gas mileage of the car in km/L. Note that 1 gal = 3.785 L.

1.5 The kinetic energy of a car of mass m moving with velocity v is $E = mv^2/2$. If $m = 1000$ kg and $v = 6$ m/s, compute E in (a) kN · m; and (b) lb · ft.

1.6 In a certain application, the coordinate a and the position coordinate x of a particle are related by

$$a = \frac{gkx}{W}$$

where g is the gravitational acceleration, k is a constant, and W is the weight of the particle. Show that this equation is dimensionally consistent if the dimension of k is $[F/L]$.

1.7 When a force F acts on a linear spring, the elongation x of the spring is given by $F = kx$, where k is called the stiffness of the spring. Determine the dimension of k in terms of the base dimensions of an absolute $[MLT]$ system of units.

1.8 In some applications dealing with very high speeds, the velocity is measured in mm/μs. Convert 8 mm/μs into (a) m/s; and (b) mi/h.

1.9 A geometry textbook gives the equation of a parabola as $y = x^2$, where x and y are measured in inches. How can this equation be dimensionally correct?

1.10 A differential equation is

$$\frac{d^2y}{dt^2} = Ay^2 + Byt$$

where y represents a distance and t is time. Determine the dimensions of constants A and B for which the equation will be dimensionally homogeneous.

1.11 The position coordinate x of a particle is determined by its velocity v and the elapsed time t as follows: (a) $x = At^2 - Bvt$; and (b) $x = Avte^{-Bt}$. Determine the dimensions of constants A and B in each case, assuming the expressions to be dimensionally correct.

1.12 A differential equation encountered in the vibration of beams is

$$\frac{d^4y}{dx^4} = \frac{\omega^2\gamma}{D}y$$

where

x = distance measured along the beam; $[x] = [L]$
y = displacement of the beam; $[y] = [L]$
ω = circular frequency of vibration; $[\omega] = [T^{-1}]$
γ = mass of the beam per unit length; $[\gamma] = [ML^{-1}]$
D = bending rigidity of beam; $[D] = [FL^2]$

Show that the equation is dimensionally homgeneous. Note that $[F] = [MLT^{-2}]$ see Eq. (1.2b).

1.13 Determine the dimensions of constants A and B for which the following equation is dimensionally homogeneous:

$$F = Akx^2 \sin\frac{Bx}{k}$$

where F is a force, x is a distance, and k represents stiffness (dimensions: $[FL^{-1}]$).

1.14 The typical power output of a compact car engine is 110 hp. What is the equivalent power in (a) lb • ft/s; and (b) kW?

1.15 Two 12-kg spheres are placed 400 mm apart. Express the gravitational attraction acting between the spheres as a percentage of their weights on earth.

1.16 Two identical spheres of radius 8 in. and weighing 2 lb on the surface of the earth are placed in contact. Find the gravitational attraction between them.

Use the following data for Problems 1.17–1.21: mass of earth $= 5.9742 \times 10^{24}$ kg, radius of earth = 6378 km, mass of moon $= 0.073\,483 \times 10^{24}$ kg, radius of moon = 1737 km.

1.17 A man weighs 170 lb on the surface of the earth. Compute his weight in an airplane flying at an elevation of 28 000 ft.

1.18 Use Eq. (1.4) to show that the weight of an object on the moon is approximately 1/6 its weight on earth.

1.19 Plot the earth's gravitational acceleration g (m/s^2) against the height h (km) above the surface of the earth.

1.20 Find the elevation h (km) where the weight of an object is one-tenth its weight on the surface of the earth.

1.21 Calculate the gravitational force between the earth and the moon in newtons. The distance between the earth and the moon is 384×10^3 km.

1.3 Fundamental Properties of Vectors

A knowledge of vectors is a prerequisite for the study of statics. In this article, we describe the fundamental properties of vectors, with subsequent articles discussing some of the more important elements of vector algebra. (The calculus of vectors will be introduced as needed in *Dynamics*.) We assume that you are already familiar with vector algebra—our discussion is intended only to be a review of the basic concepts.

The differences between scalar and vector quantities must be understood:

> A *scalar* is a quantity that has magnitude only. A *vector* is a quantity that possesses magnitude and direction and obeys the parallelogram law for addition.

Because scalars possess only magnitudes, they are real numbers that can be positive, negative, or zero. Physical quantities that are scalars include temperature, time, and speed. As shown later, force, velocity, and displacement are examples of physical quantities that are vectors. The magnitude of a vector is always taken to be a nonnegative number. When a vector represents a physical quantity, the units of the vector are taken to be the same as the units of its magnitude (pounds, meters per second, feet, etc.).

The algebraic notation used for a scalar quantity must, of course, be different from that used for a vector quantity. In this text, we adopt the following conventions: (1) scalars are written as italicized English or Greek letters—for example, t for time and θ for angle; (2) vectors are written as boldface letters—for example, **F** for force; and (3) the magnitude of a vector **A** is denoted as $|\mathbf{A}|$ or simply as A (italic).

There is no universal method for indicating vector quantities when writing by hand. The more common notations are $\vec{A}$, $\underset{\rightarrow}{A}$, $\bar{A}$, and $\underline{A}$. Unless instructed otherwise, you are free to use the convention that you find most comfortable. However, it is imperative that you take care to always distinguish between scalars and vectors when you write.

The following summarizes several important properties of vectors.

Vectors as Directed Line Segments Any vector **A** can be represented geometrically as a directed line segment (an arrow), as shown in Fig. 1.2(a). The magnitude of **A** is denoted by A, and the direction of **A** is specified by the sense of the arrow and the angle θ that it makes with a fixed reference line. When using graphical methods, the length of the arrow is drawn proportional to the magnitude of the vector. Observe that the representation shown in Fig. 1.2(a) is complete because both the magnitude and direction of the vector are indicated. In some instances, it is also convenient to use the representation shown in Fig. 1.2(b), where the vector character of **A** is given

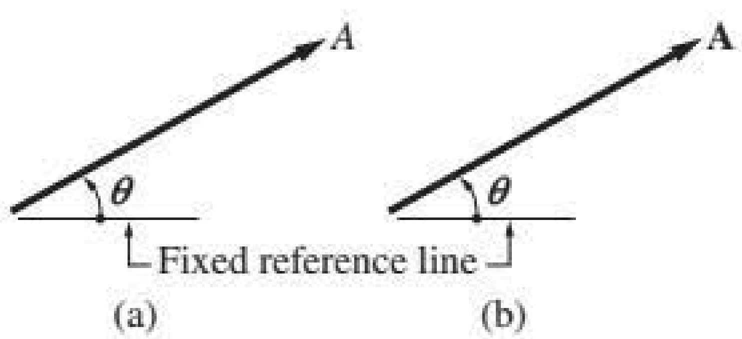


Figure 1.2

additional emphasis by using boldface. Both of these representations for vectors are used in this text.

We see that a vector does not possess a unique line of action, because moving a vector to a parallel line of action changes neither its magnitude nor its direction. In some engineering applications, the definition of a vector is more restrictive to include a line of action or even a point of application—see Art. 2.2.

Equality of Vectors Two vectors **A** and **B** are said to be equal, written as $\mathbf{A} = \mathbf{B}$, if (1) their magnitudes are equal—that is, $A = B$, and (2) they have the same direction.

Scalar-Vector Multiplication The multiplication of a scalar m and a vector **A**, written as $m\mathbf{A}$ or as $\mathbf{A}m$, is defined as follows.

1. If m is positive, $m\mathbf{A}$ is the vector of magnitude mA that has the same direction as **A**.
2. If m is negative, $m\mathbf{A}$ is the vector of magnitude $|m|A$ that is oppositely directed to **A**.
3. If $m = 0$, $m\mathbf{A}$ (called the null or zero vector) is a vector of zero magnitude and arbitrary direction.

For $m = -1$, we see that $(-1)\mathbf{A}$ is the vector that has the same magnitude as **A** but is oppositely directed to **A**. The vector $(-1)\mathbf{A}$, usually written as $-\mathbf{A}$, is called the *negative of* **A**.

Unit Vectors A unit vector is a dimensionless vector with magnitude 1. Therefore, if $\boldsymbol{\lambda}$ represents a unit vector ($|\boldsymbol{\lambda}| = 1$) with the same direction as **A**, we can write

$$\mathbf{A} = A\boldsymbol{\lambda}$$

This representation of a vector often is useful because it separates the magnitude A and the direction $\boldsymbol{\lambda}$ of the vector.

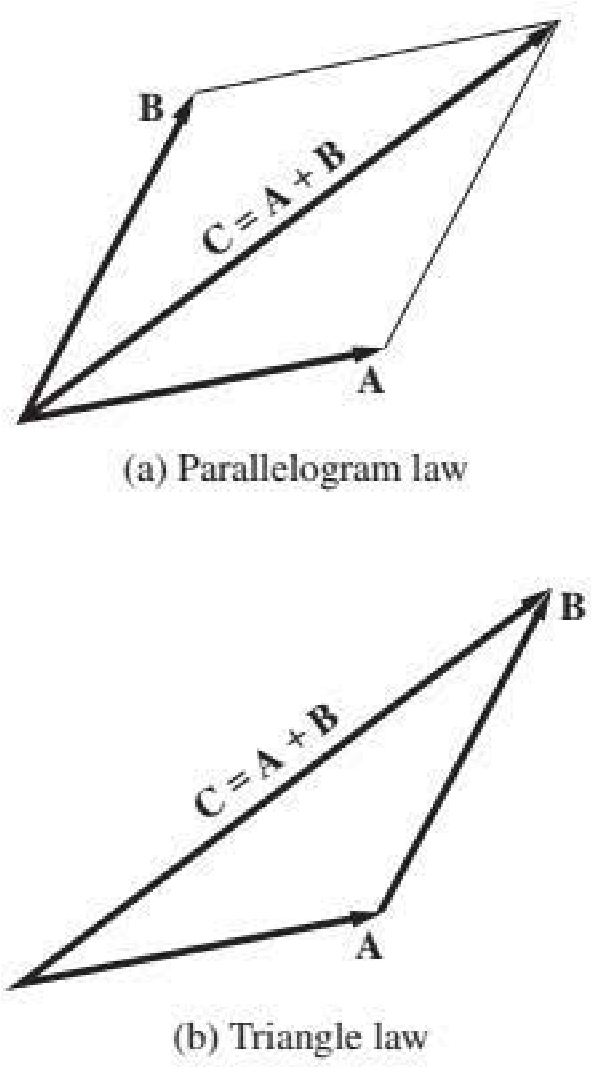


Figure 1.3

The Parallelogram Law for Addition and the Triangle Law The addition of two vectors **A** and **B** is defined to be the vector **C** that results from the geometric construction shown in Fig. 1.3(a). Observe that **C** is the diagonal of the parallelogram formed by **A** and **B**. The operation depicted in Fig. 1.3(a), written as $\mathbf{A} + \mathbf{B} = \mathbf{C}$, is called the *parallelogram law for addition*. The vectors **A** and **B** are referred to as *components* of **C**, and **C** is called the *resultant* of **A** and **B**. The process of replacing a resultant with its components is called *resolution*. For example, **C** in Fig. 1.3(a) is resolved into its components **A** and **B**.

An equivalent statement of the parallelogram law is the *triangle law*, which is shown in Fig. 1.3(b). Here the tail of **B** is placed at the tip of **A**, and **C** is the vector that completes the triangle, drawn from the tail of **A** to the tip of **B**. The result is identical if the tail of **A** is placed at the tip of **B** and **C** is drawn from the tail of **B** to the tip of **A**.

Letting **E**, **F**, and **G** represent any three vectors, we have the following two important properties (each follows directly from the parallelogram law):

- Addition is commutative: **E** + **F** = **F** + **E**
- Addition is associative: **E** + (**F** + **G**) = (**E** + **F**) + **G**

It is often convenient to find the sum **E** + **F** + **G** (no parentheses are needed) by adding the vectors from tip to tail, as shown in Fig. 1.4. The sum of the three vectors is seen to be the vector drawn from the tail of the first vector (**E**) to the tip of the last vector (**G**). This method, called the *polygon rule for addition*, can easily be extended to any number of vectors.

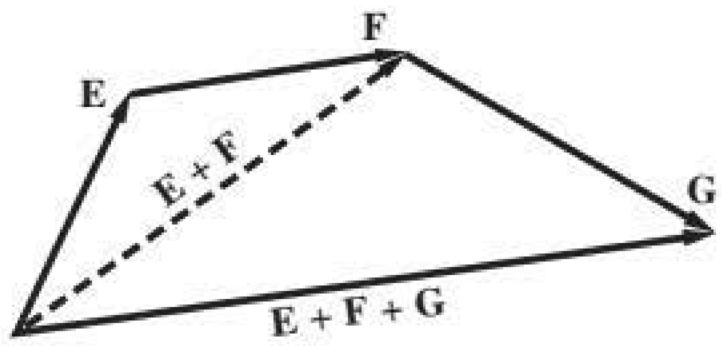


Figure 1.4

The subtraction of two vectors **A** and **B**, written as **A** − **B**, is defined as **A** − **B** = **A** + (−**B**), as shown in Fig. 1.5.

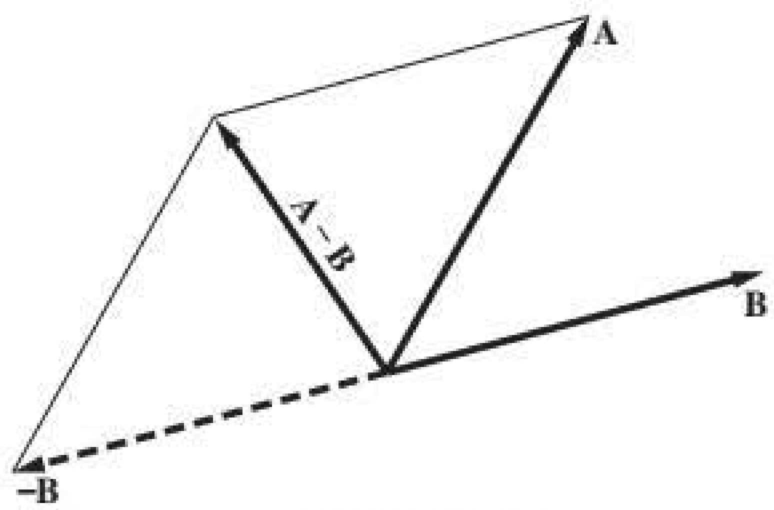


Figure 1.5

Because of the geometric nature of the parallelogram law and the triangle law, vector addition can be accomplished graphically. A second technique is to determine the relationships between the various magnitudes and angles analytically by applying the *laws of sines and cosines* to a sketch of the parallelogram (or the triangle)—see Table 1.1. Both the graphical and the analytical methods are illustrated in Sample Problem 1.4.

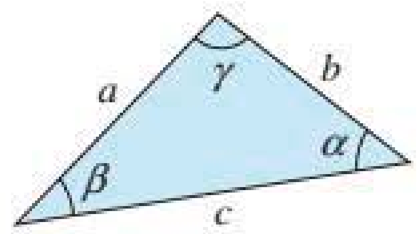


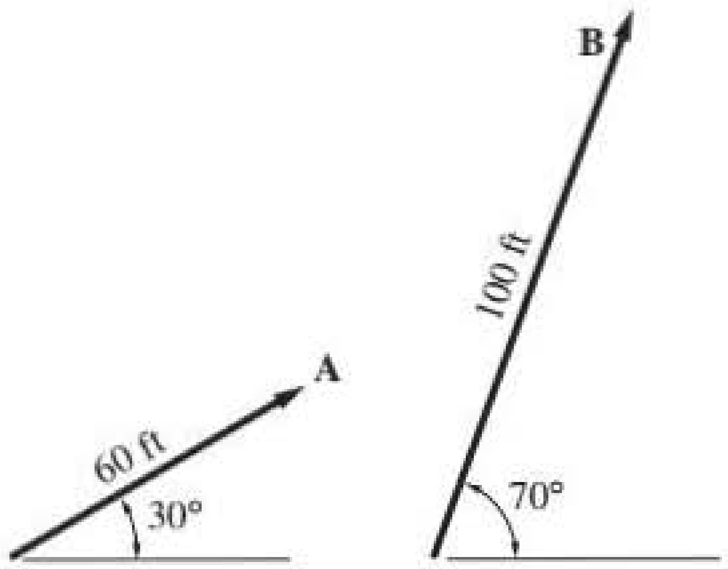
Table 1.1

Law of sines	$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$
Law of cosines	$a^2 = b^2 + c^2 - 2bc\cos\alpha$ $b^2 = c^2 + a^2 - 2ca\cos\beta$ $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Some words of caution: It is unfortunate that the symbols +, −, and = are commonly used in both scalar algebra and vector algebra, because they have completely different meanings in the two systems. For example, note the different meanings for + and = in the following two equations: $\mathbf{A} + \mathbf{B} = \mathbf{C}$ and $1 + 2 = 3$. In computer programming, this is known as *operator overloading*, where the rules of the operation depend on the operands involved in the process. Unless you are extremely careful, this double meaning for symbols can easily lead to invalid expressions—for example, $\mathbf{A} + 5$ (a vector cannot be added to a scalar!) and $\mathbf{A} = 1$ (a vector cannot equal a scalar!).

Sample Problem 1.4

Figure (a) shows two position vectors of magnitudes $A = 60$ ft and $B = 100$ ft. (A position vector is a vector drawn between two points in space.) Determine the resultant $\mathbf{R} = \mathbf{A} + \mathbf{B}$ using the following methods: (1) analytically, using the triangle law; and (2) graphically, using the triangle law.



(a)

Solution

Part 1

The first step in the analytical solution is to draw a sketch (approximately to scale) of the triangle law. The magnitude and direction of the resultant are then found by applying the laws of sines and cosines to the triangle.

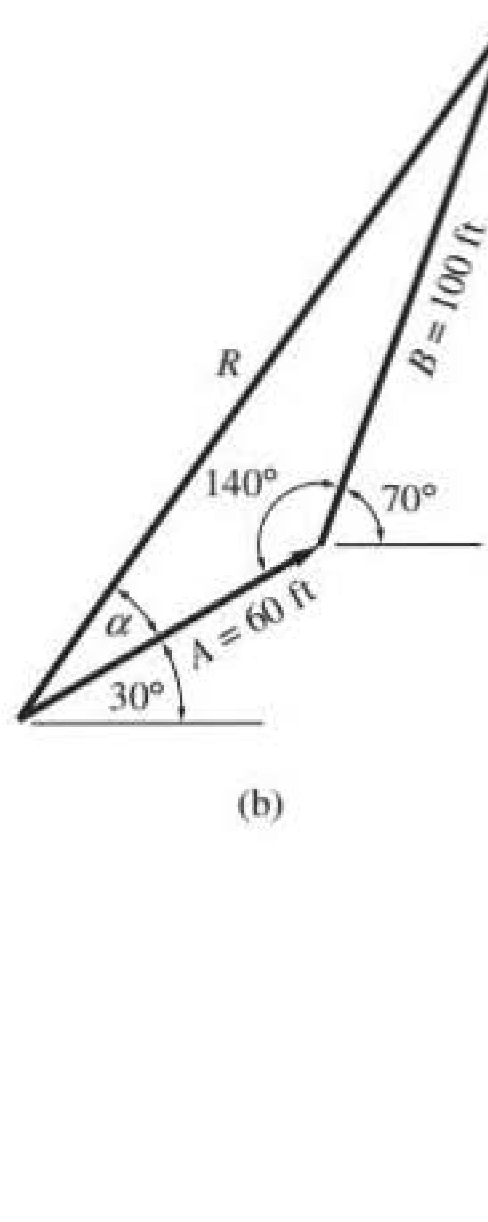
In this problem, the triangle law for the vector addition of **A** and **B** is shown in Fig. (b). The magnitude R of the resultant and the angle α are the unknowns to be determined. Applying the law of cosines, we obtain

$$R^2 = 60^2 + 100^2 - 2(60)(100)\cos 140°$$

which yields $R = 151.0$ ft.

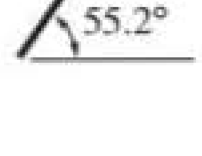
The angle α can now be found from the law of sines:

$$\frac{100}{\sin\alpha} = \frac{R}{\sin 140°}$$



(b)

Substituting $R = 151.0$ ft and solving for α, we get $\alpha = 25.2°$. Referring to Fig. (b), we see that the angle that **R** makes with the horizontal is $30° + \alpha = 30° + 25.2° = 55.2°$. Therefore, the resultant of **A** and **B** is



Answer

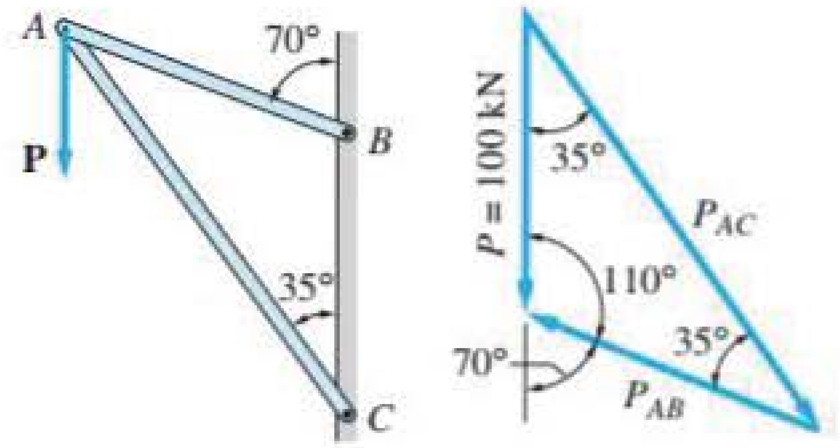
Part 2

In the graphical solution, Fig. (b) is drawn to scale with the aid of a ruler and a protractor. We first draw the vector **A** at 30° to the horizontal and then append vector **B** at 70° to the horizontal. The resultant **R** is then obtained by drawing a line from the tail of **A** to the head of **B**. The magnitude of **R** and the angle it makes with the horizontal can now be measured directly from the figure.

Of course, the results would not be as accurate as those obtained in the analytical solution. If care is taken in making the drawing, two-digit accuracy is the best we can hope for. In this problem we should get $R \approx 150$ ft, inclined at 55° to the horizontal.

Sample Problem 1.5

The vertical force **P** of magnitude 100 kN is applied to the frame shown in Fig. (a). Resolve **P** into components that are parallel to the members AB and AC of the truss.



(a) (b)

Solution

The force triangle in Fig. (b) represents the vector addition $\mathbf{P} = \mathbf{P}_{AC} + \mathbf{P}_{AB}$. The angles in the figure were derived from the inclinations of AC and AB with the vertical: $\mathbf{P}_{AC}$ is inclined at 35° (parallel to AC), and $\mathbf{P}_{AB}$ is inclined at 70° (parallel to AB). Applying the law of sines to the triangle, we obtain

$$\frac{100}{\sin 35°} = \frac{P_{AB}}{\sin 35°} = \frac{P_{AC}}{\sin 110°}$$

which yields for the magnitudes of the components

$$P_{AB} = 100.0 \text{ kN} \qquad P_{BC} = 163.8 \text{ kN}$$ **Answer**